

mines the secular changes of each of the *elements* or determining quantities of the orbit. In 1773, Laplace also attacked this subject of secular changes, and obtained expressions for them. On this occasion, he proved the celebrated proposition that, "the mean motions of the planets are invariable:" that is, that there is, in the revolutions of the system, no progressive change which is not finally stopped and reversed; no increase, which is not, after some period, changed into decrease; no retardation which is not at last succeeded by acceleration; although, in some cases, millions of years may elapse before the system reaches the turning-point. Thomas Simpson noticed the same consequence of the laws of universal attraction. In 1774 and 1776, Lagrange<sup>6</sup> still labored at the secular equations; extending his researches to the nodes and inclinations; and showed that the invariability of the mean motions of the planets, which Laplace had proved, neglecting the fourth powers of the eccentricities and inclinations of the orbits,<sup>7</sup> was true, however far the approximation was carried, so long as the squares of the disturbing masses were neglected. He afterwards improved his methods;<sup>8</sup> and, in 1783, he endeavored to extend the calculation of the changes of the elements to the periodical equations, as well as the secular.

8. *Mécanique Céleste, &c.*—Laplace also resumed the consideration of the secular changes; and, finally, undertook his vast work, the *Mécanique Céleste*, which he intended to contain a complete view of the existing state of this splendid department of science. We may see, in the exultation which the author obviously feels at the thought of erecting this monument of his age, the enthusiasm which had been excited by the splendid course of mathematical successes of which I have given a sketch. The two first volumes of this great work appeared in 1799. The third and fourth volumes were published in 1802 and 1805 respectively. Since its publication, little has been added to the solution of the great problems of which it treats. In 1808, Laplace presented to the French Bureau des Longitudes, a Supplement to the *Mécanique Céleste*; the object of which was to improve still further

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says, "It consists in regarding the elements of the elliptical motion as variable in virtue of the perturbing forces. Those elements are, 1, the axis major; 2, the epoch of the body being at the apse; 3, the eccentricity; 4, the movement of the apse; 5, the inclination; 6, the longitude of the node;" and he then proceeds to show how Euler did this. It is possible that Lagrange knew nothing of Euler's paper. See *Méc. Cél.* vol. v. p. 812. But Euler's conception and treatment of the method are complete, so that he must be looked upon as the author of it.

<sup>6</sup> Gautier, p. 104.

<sup>7</sup> *Ib.* p. 184.

<sup>8</sup> *Ib.* p. 196.