the mode of obtaining the secular variations of the elements. Poisson and Lagrange proved the invariability of the major axes of the orbits, as far as the second order of the perturbing forces. Various other authors have since labored at this subject. Burckhardt, in 1808, extended the perturbing function as far as the sixth order of the eccentricities. Gauss, Hansen, and Bessel, Ivory, MM. Lubbock, Plana, Pontécoulant, and Airy, have, at different periods up to the present time, either extended or illustrated some particular part of the theory, or applied it to special cases; as in the instance of Professor Airy's calculation of an inequality of Venus and the earth, of which the period is 240 years. The approximation of the Moon's motions has been pushed to an almost incredible extent by M. Damoiseau, and, finally, Plana has once more attempted to present, in a single work (three thick quarto volumes), all that has hitherto been executed with regard to the theory of the Moon.

I give only the leading points of the progress of analytical dynamics. Hence I have not spoken in detail of the theory of the Satellites of Jupiter, a subject on which Lagrange gained a prize for a Memoir, in 1766, and in which Laplace discovered some most curious properties in 1784. Still less have I referred to the purely speculative question of *Tautochronous Curves* in a resisting medium, though it was a subject of the labors of Bernoulli, Euler, Fontaine, D'Alembert, Lagrange, and Laplace. The reader will rightly suppose that many other curious investigations are passed over in utter silence.

[2d Ed.] [Although the analytical calculations of the great mathematicians of the last century had determined, in a demonstrative manner, a vast series of inequalities to which the motions of the sun, moon, and planets were subject in virtue of their mutual attraction, there were still unsatisfactory points in the solutions thus given of the great mechanical problems suggested by the System of the Universe. One of these points was the want of any evident mechanical significance in the successive members of these series. Lindenau relates that Lagrange, near the end of his life, expressed his sorrow that the methods of approximation employed in Physical Astronomy rested on arbitrary processes, and not on any insight into the results of mechanical action. But something was subsequently done to remove the ground of this complaint. In 1818, Gauss pointed out that secular equations may be conceived to result from the disturbing body being distributed along its orbit so as to form a ring, and thus made the result conceivable more distinctly than as a mere result of calculation. And it appears