

ponding Inequalities in the motions of the Satellites of other planets, arising from the same cause; and likewise pointed out the necessary existence of irregularities in the motions of the Planets arising from their mutual attraction. Newton gave propositions by which the Irregularities of the motion of Jupiter's moons might be deduced from those of our own;¹⁴ and it was shown that the motions of their nodes would be slow by theory, as Flamsteed had found it to be by observation.¹⁵ But Newton did not attempt to calculate the effect of the mutual action of the planets, though he observes, that in the case of Jupiter and Saturn this effect is too considerable to be neglected;¹⁶ and he notices in the second edition,¹⁷ that it follows from the theory of gravity, that the aphelia of Mercury, Venus, the Earth, and Mars, slightly progress.

In one celebrated instance, indeed, the deviation of the theory of the *Principia* from observation was wider, and more difficult to explain; and as this deviation for a time resisted the analysis of Euler and Clairaut, as it had resisted the synthesis of Newton, it at one period staggered the faith of mathematicians in the exactness of the law of the inverse square of the distance. I speak of the Motion of the Moon's Apogee, a problem which has already been referred to; and in which Newton's method, and all the methods which could be devised for some time afterwards, gave only half the observed motion; a circumstance which arose, as was discovered by Clairaut in 1750, from the insufficiency of the method of approximation. Newton does not attempt to conceal this discrepancy. After calculating what the motion of apse would be, upon the assumption of a disturbing force of the same amount as that which the sun exerts on the moon, he simply says,¹⁸ "the apse of the moon moves about twice as fast."

The difficulty of doing what Newton did in this branch of the subject, and the powers it must have required, may be judged of from what has already been stated;—that no one, with his methods, has yet been able to add any thing to his labors: few have undertaken to illustrate what he has written, and no great number have understood it throughout. The extreme complication of the forces, and of the conditions under which they act, makes the subject by far the most thorny walk of mathematics. It is necessary to resolve the action

¹⁴ B. i. Prop. 66.

¹⁵ B. iii. Prop. 28.

¹⁶ B. iii. Prop. 18.

¹⁷ Scholium to Prop. 14. B. iii.

¹⁸ B. i. Prop. 44, second edit. There is reason to believe, however, that Newton had, in his unpublished calculations, rectified this discrepancy.