had noticed[1] that when a string vibrates, one which is in unison with it vibrates without being touched. He was also aware that this was true if the second string was an octave or a twelfth below the first. This was observed as a new fact in England in 1674, and communicated to the Royal Society by Wallis.[2] But the later observers ascertained further, that the longer string divides itself into two, or into three equal parts, separated by *nodes*, or points of rest; this they proved by hanging bits of paper on different parts of the string. The discovery so modified was again made by Sauveur[3] about 1700. The sounds thus produced in one string by the vibration of another, have been termed *Sympathetic Sounds*. Similar sounds are often produced by performers on stringed instruments, by touching the string at one of its aliquot divisions, and are then called the *Acute Harmonics*. Such facts were not difficult to explain on Taylor's view of the mechanical condition of the string; but the difficulty was increased when it was noticed that a sounding body could produce these different notes *at the same time*. Mersenne had remarked this, and the fact was more distinctly observed and pursued by Sauveur. The notes thus produced in addition to the genuine note of the string, have been called *Secondary Notes;* those usually heard are, the Octave, the Twelfth, and the Seventeenth above the note itself. To supply a mode of conceiving distinctly, and explaining mechanically, vibrations which should allow of such an effect, was therefore a requisite step in acoustics.

This task was performed by Daniel Bernoulli in a memoir published in 1755.[4] He there stated and proved the Principle of *the coexistence of small vibrations*. It was already established, that a string might vibrate either in a single *swelling* (if we use this word to express the curve between two nodes which Bernoulli calls a *ventre*), or in two or three or any number of equal swellings with immoveable nodes between. Daniel Bernoulli showed further, that these nodes might be combined, each taking place as if it were the only one. This appears sufficient to explain the coexistence of the harmonic sounds just noticed. D'Alembert, indeed, in the article *Fundamental* in the French *Encyclopédie*, and Lagrange in his *Dissertation on Sound* in the *Turin Memoirs*,[5] offer several objections to this explanation; and it cannot be denied that the subject has its difficulties; but

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[1] *Harm.* lib. iv. Prop. 28 (1636). [2] *Ph. Tr.* 1677, April. [3] *A. P.* 1701.
[4] *Berlin Mem.* 1753, p. 147. [5] T. i. pp. 64, 103.