

the vexed questions which then exercised the minds of thinkers, was not reached by a detailed psychological investigation such as has since been carried out through the labours of independent thinkers in all the three countries, *e.g.*, Mill, Renouvier, Wundt, and their successors; it was gained by a much shorter and much more abstract process. Kant relied on two points which he considered were well established. The first and most important of these was the existence of a definite amount of perfectly certain and assured knowledge contained in the sciences of mathematics and mathematical physics; the second was a definite body of doctrine contained in the formal logic and the empirical psychology of the schools, both of which Kant himself taught in his academic courses. So far as the first point is concerned, Kant had a broader foundation to build on than Descartes before him, inasmuch as he could not only point to pure mathematics, but had in addition also, what he considered the ideal of scientific achievement—the natural philosophy of Newton.<sup>1</sup> So far as the second point is

<sup>1</sup> It has, however, been shown (*e.g.*, by E. Dühring in his 'Kritische Geschichte der Allgemeinen Principien der Mechanik,' 3rd ed., 1887) that Kant's notions as to the principles of dynamics and physics were still extremely inaccurate and confused. Although in the minds of some of the great mathematicians, such as Newton in England and d'Alembert in France, very precise views existed, these have only very slowly become the property of philosophical thinkers. Nor does it appear as if Kant himself contributed much to this important clearance of ideas. Neither his

early tract, which deals with the measure of *vis viva* (1753), nor his treatment of dynamical and physical conceptions in the celebrated 'Natural History of the Heavens' (1755), shows any strict definition or consistent use of dynamical principles. And it is significant that Ernst Mach in his historical Treatise on these subjects ('Mechanik in ihrer Entwicklung,' 1883, Eng. trans. by M'Cormick) has no occasion to refer to Kant. With Kant the fundamental notions of arithmetic (numerical and general), of geometry (synthetic and analytic), of