

length over parts of science that had been abandoned in despair, and given over to hopeless obscurity.

(185.) The verification of *quantitative* laws has been already spoken of (178.); but their importance in physical science is so very great, inasmuch as they alone afford a handle to strict mathematical deductive application, that something ought to be said of the nature of the inductions by which they are to be arrived at. In their simplest or least general stages (of which alone we speak at present), they usually express some numerical relation between two quantities dependent on each other, either as collateral effects of a common cause, or as the amount of its effect under given numerical circumstances or *data*. For example, the law of refraction before noticed (§ 22.) expresses, by a very simple relation, the amount of angular deviation of a ray of light from its course, when the *angle* at which it is inclined to the refracting surface is known, viz. that the *sine* of the angle which the incident ray makes with a perpendicular to the surface is always to that of the angle made by the refracted ray with the same perpendicular, in a constant proportion, so long as the refracting substance is the same. To arrive inductively at laws of this kind, where one quantity *depends on* or *varies with* another, all that is required is a series of careful and exact measures in every different state of the *datum* and *quæsitum*. Here, however, the mathematical form of the law being of the highest importance, the greatest attention must be given to the *extreme cases* as well as to all those points where the one quantity changes rapidly with a small change of the other.* The results must be set down in a table in which the *datum* gradually increases in magnitude from the lowest to the highest limit of which it is susceptible. It will depend, then, entirely on our habit of treating mathematical subjects, how far we may be able to include such a table in the

* A very curious instance of the pursuit of a law completely empirical into an extreme case is to be found in Newton's rule for the dilatation of his colored rings seen between glasses at great obliquities. Optics, book ii. part i. obs. 7.