other things being equal, not as the weight, but as the square root of the weight. But he is right as to the essential point, that those ratios of 2 to 1, 3 to 2, and 4 to 3, are the characteristic ratios of the Octave, Fifth, and Fourth. In order to produce these intervals, the appended weights must be, not as 12, 9, 8, and 6, but as 12, $6\frac{3}{4}$, $5\frac{1}{3}$, and 3.

The numerical relations of the other intervals of the musical scale, as well as of the Octave, Fifth, and Fourth, were discovered by the Greeks. Thus they found that the proportion in a Major Third was 5 to 4; in a Minor Third, 6 to 5; in a Major Tone, 9 to 8; in a Semitone or *Diesis*, 16 to 15. They even went so far as to determine the *Comma*, in which the interval of two notes is so small that they are in the proportion of 81 to 80. This is the interval between two notes, each of which may be called the Seventeenth above the key-note;—the one note being obtained by ascending a Fifth four times over; the other being obtained by ascending through two Octaves and a Major Third. The want of exact coincidence between these two notes is an inherent arithmetical imperfection in the musical scale, of which the consequences are very extensive.

The numerical properties of the musical scale were worked out to a very great extent by the Greeks, and many of their Treatises on this subject remain to us. The principal ones are the seven authors published by Meibomius.[1] These arithmetical elements of Music are to the present day important and fundamental portions of the Science of Harmonics.

It may at first appear that the truth, or even the possibility of this history, by referring the discovery to accident, disproves our doctrine, that this, like all other fundamental discoveries, required a distinct and well-pondered Idea as its condition. In this, however, as in all cases of supposed accidental discoveries in science, it will be found, that it was exactly the possession of such an Idea which made the accident possible.

Pythagoras, assuming the truth of the tradition, must have had an exact and ready apprehension of those relations of musical sounds, which are called respectively an Octave, a Fifth, and a Fourth. If he had not been able to conceive distinctly this relation, and to apprehend it when heard, the sounds of the anvil would have struck his ears to no more purpose than they did those of the smiths themselves. He

[1] *Antiquæ Musicæ Scriptores septem*, 1652.