

is, take the cube root of it, and double the proportion so found, that is, square it, he will find the exact proportion of the distances of the Earth and of Saturn from the sun. For the cube root of 1 is 1, and the square of this is 1; and the cube root of 30 is greater than 3, and therefore the square of it is greater than 9. And Saturn at his mean distance from the sun is at a little more than 9 times the mean distance of the Earth."

When we now look back at the time and exertions which the establishment of this law cost Kepler, we are tempted to imagine that he was strangely blind in not seeing it sooner. His object, we might reason, was to discover a law connecting the distances and the periodic times. What law of connection could be more simple and obvious, we might say, than that one of these quantities should vary as some *power* of the other, or as some *root*; or as some combination of the two, which in a more general view, may still be called a *power*? And if the problem had been viewed in this way, the question must have occurred, to *what* power of the periodic times are the distances proportional? And the answer must have been, the trial being made, that they are proportional to the square of the cube root. This *ex-post-facto* obviousness of discoveries is a delusion to which we are liable with regard to many of the most important principles. In the case of Kepler, we may observe, that the process of connecting two classes of quantities by comparing their *powers*, is obvious only to those who are familiar with general algebraical views; and that in Kepler's time, algebra had not taken the place of geometry, as the most usual vehicle of mathematical reasoning. It may be added, also, that Kepler always sought his *formal* laws by means of *physical* reasonings; and these, though vague or erroneous, determined the nature of the mathematical connection which he assumed. Thus in the *Mysterium* he had been led by his notions of moving virtue of the sun to this conjecture, among others—that, in the planets, the increase of the periods will be double of the difference of the distances; which supposition he found to give him an approach to the actual proportion of the distances, but one not sufficiently close to satisfy him.

The greater part of the fifth Book of the *Harmonics of the Universe* consists in attempts to explain various relations among the distances, times, and eccentricities of the planets, by means of the ratios which belong to certain concords and discords. This portion of the work is so complex and laborious, that probably few modern readers have had courage to go through it. Delambre acknowledged that his patience