

3. The bisection of the eccentricity, and a stationary point of equations, after the manner of Ptolemy.

4. The vicarious hypothesis by a free section of the eccentricity made to agree as nearly as possible with the truth.

5. The physical hypothesis on the supposition of a perfect circle.

6. The physical hypothesis on the supposition of a perfect ellipse.

By the physical hypothesis, he meant the doctrine that the time of a planet's describing any part of its orbit is proportional to the distance of the planet from the sun, for which supposition, as we have said, he conceived that he had assigned physical reasons.

The two last hypotheses came the nearest to the truth, and differed from it only by about eight minutes, the one in excess and the other in defect. And, after being much perplexed by this remaining error, it at last occurred to him¹² that he might take another ellipsis, exactly intermediate between the former one and the circle, and that this must give the path and the motion of the planet. Making this assumption, and taking the areas to represent the times, he now saw¹³ that both the longitude and the distances of Mars would agree with observation to the requisite degree of accuracy. The rectification of the former hypothesis, when thus stated, may, perhaps, appear obvious. And Kepler informs us that he had nearly been anticipated in this step (c. 55). "David Fabricius, to whom I had communicated my hypothesis of cap. 45, was able, by his observations, to show that it erred in making the distances too short at mean longitudes; of which he informed me by letter while I was laboring, by repeated efforts, to discover the true hypothesis. So nearly did he get the start of me in detecting the truth." But this was less easy than it might seem. When Kepler's first hypothesis was enveloped in the complex construction requisite in order to apply it to each point of the orbit, it was far more difficult to see where the error lay, and Kepler hit upon it only by noticing the coincidences of certain numbers, which, as he says, raised him as if from sleep, and gave him a new light. We may observe, also, that he was perplexed to reconcile this new view, according to which the planet described an exact ellipse, with his former opinion, which represented the motion by means of libration in an epicycle. "This," he says, "was my greatest trouble, that, though I considered and reflected till I was almost mad, I could not find why the planet to which, with so much probability, and with such an exact

¹² *De Stellâ Martis*, c. 58.

¹³ *Ibid.* p. 285.