http://www.geology.19thcenturyscience.org/books/1858-Whewell-HistSci/README.htm

surface: but he erroneously judged the velocity to be exactly proportional to the depth. Torricelli found that the fluid, under the inevitable causes of defect which occur in the experiment, would spout nearly to the height of the surface: he therefore inferred, that the full velocity is that which a body would acquire in falling through the depth; and that it is consequently proportional to the square root of the depth.—This, however, he stated only as a result of experience, or law of phenomena, at the end of his treatise, *De Motu Naturaliter Accelerato*, printed in 1643.

Newton treated the subject theoretically in the *Principia* (1687); but we must allow, as Lagrange says, that this is the least satisfactory passage of that great work. Newton, having made his experiments in another manner than Torricelli, namely, by measuring the *quantity* of the efflux instead of its velocity, found a result inconsistent with that of Torricelli. The velocity inferred from the quantity discharged, was only that due to *half* the depth of the fluid.

In the first edition of the *Principia*,[3] Newton gave a train of reasoning by which he theoretically demonstrated his own result, going upon the principle, that the momentum of the issuing fluid is equal to the momentum which the column vertically over the orifice would generate by its gravity. But Torricelli's experiments, which had given the velocity due to the whole depth, were confirmed on repetition: how was this discrepancy to be explained?

Newton explained the discrepancy by observing the contraction which the jet, or vein of water, undergoes, just after it leaves the orifice, and which he called the *vena contracta*. At the orifice, the velocity is that due to half the height; at the vena contracta it is that due to the whole height. The former velocity regulates the quantity of the discharge; the latter, the path of the jet.

This explanation was an important step in the subject; but it made Newton's original proof appear very defective, to say the least. In the second edition of the *Principia* (1714), Newton attacked the problem in a manner altogether different from his former investigation. He there assumed, that when a round vessel, containing fluid, has a hole in its bottom, the descending fluid may be conceived to be a conoidal mass, which has its base at the surface of the fluid, and its narrow end at the orifice. This portion of the fluid he calls the *cataract;* and supposes that while this part descends, the surrounding

[3] B. ii. Prop. xxxvii.