

CHAPTER V.

GENERALIZATION OF THE PRINCIPLES OF MECHANICS.

Sect. 1.—Generalization of the Second Law of Motion.—Central Forces.

THE Second Law of Motion being proved for constant Forces which act in parallel lines, and the Third Law for the Direct Action of bodies, it still required great mathematical talent, and some inductive power, to see clearly the laws which govern the motion of any number of bodies, acted upon by each other, and by any forces, anyhow varying in magnitude and direction. This was the task of the generalization of the laws of motion.

Galileo had convinced himself that the velocity of projection, and that which gravity alone would produce, are “both maintained, without being altered, perturbed, or impeded in their mixture.” It is to be observed, however, that the truth of this result depends upon a particular circumstance, namely, that gravity, at all points, acts in lines, which, as to sense, are parallel. When we have to consider cases in which this is not true, as when the force tends to the centre of a circle, the law of composition cannot be applied in the same way; and, in this case, mathematicians were met by some peculiar difficulties.

One of these difficulties arises from the apparent inconsistency of the statical and dynamical measures of force. When a body moves in a circle, the force which urges the body to the centre is only a *tendency* to motion; for the body does not, in fact, approach to the centre; and this mere tendency to motion is combined with an actual motion, which takes place in the circumference. We appear to have to compare two things which are heterogeneous. Descartes had noticed this difficulty, but without giving any satisfactory solution of it.¹ If we combine the actual motion to or from the centre with the traverse motion about the centre, we obtain a result which is false on mechanical principles. Galileo endeavored in this way to find the curve described by a body which falls towards the earth’s centre, and is, at the same time, carried

¹ *Princip.* P. iii. 59.