

tions which succeed in the Lunar Theory fail here. Artifices somewhat modified are required to overcome the difficulties of this case.

Euler had investigated, in particular, the motions of Jupiter and Saturn, in which there was a secular acceleration and retardation, known by observation, but not easily explicable by theory. Euler's memoirs, which gained the prize of the French Academy, in 1748 and 1752, contained much beautiful analysis; and Lagrange published also a theory of Jupiter and Saturn, in which he obtained results different from those of Euler. Laplace, in 1787, showed that this inequality arose from the circumstance that two of Saturn's years are very nearly equal to five of Jupiter's.

The problems relating to Jupiter's *Satellites*, were found to be even more complex than those which refer to the planets: for it was necessary to consider each satellite as disturbed by the other three at once; and thus there occurred the Problem of *Five Bodies*. This problem was resolved by Lagrange.<sup>3</sup>

Again, the newly-discovered *small Planets*, Juno, Ceres, Vesta, Pallas, whose orbits almost coincide with each other, and are more inclined and more eccentric than those of the ancient planets, give rise, by their perturbations, to new forms of the problem, and require new artifices.

In the course of these researches respecting Jupiter, Lagrange and Laplace were led to consider particularly the *secular Inequalities* of the solar system; that is, those inequalities in which the duration of the cycle of change embraces very many revolutions of the bodies themselves. Euler in 1749 and 1755, and Lagrange<sup>4</sup> in 1766, had introduced the method of the *Variation of the Elements* of the orbit; which consists in tracing the effect of the perturbing forces, not as directly altering the place of the planet, but as producing a change from one instant to another, in the dimensions and position of the Elliptical orbit which the planet describes.<sup>5</sup> Taking this view, he deter-

<sup>3</sup> Bailly, *Ast. Mod.* iii. 178.

<sup>4</sup> Gautier, *Prob. de Trois Corps*, p. 155.

<sup>5</sup> In the first edition of this History, I had ascribed to Lagrange the invention of the Method of Variation of Elements in the theory of Perturbations. But justice to Euler requires that we should assign this distinction to him; at least, next to Newton, whose mode of representing the paths of bodies by means of a *Revolving Orbit*, in the Ninth Section of the *Principia*, may be considered as an anticipation of the method of variation of elements. In the fifth volume of the *Mécanique Céleste*, livre xv. p. 805, is an abstract of Euler's paper of 1749; where Laplace adds, "C'est le premier essai de la méthode de la variation des constantes arbitraires." And in page 810 is an abstract of the paper of 1766: and speaking of the method, Laplace