

1748, Euler not only assented to the generalization of D'Alembert, but held that it was not necessary that the curves so introduced should be defined by any algebraical condition whatever. From this extreme indeterminateness D'Alembert dissented; while Daniel Bernoulli, trusting more to physical and less to analytical reasonings, maintained that both these generalizations were inapplicable in fact, and that the solution was really restricted, as had at first been supposed, to the form of the trochoid, and to other forms derivable from that. He introduced, in such problems, the "Law of Coexistent Vibrations," which is of eminent use in enabling us to conceive the results of complex mechanical conditions, and the real import of many analytical expressions. In the mean time, the wonderful analytical genius of Lagrange had applied itself to this problem. He had formed the Academy of Turin, in conjunction with his friends Saluces and Cigna; and the first memoir in their Transactions was one by him on this subject: in this and in subsequent writings he has established, to the satisfaction of the mathematical world, that the functions introduced in such cases are not necessarily continuous, but are arbitrary to the same degree that the motion is so practically; though capable of expression by a series of circular functions. This controversy, concerning the degree of lawlessness with which the conditions of the solution may be assumed, is of consequence, not only with respect to vibrating strings, but also with respect to many problems, belonging to a branch of Mechanics which we now have to mention, the Doctrine of Fluids.

11. *Equilibrium of Fluids. Figure of the Earth. Tides.*—The application of the general doctrines of Mechanics to fluids was a natural and inevitable step, when the principles of the science had been generalized. It was easily seen that a fluid is, for this purpose, nothing more than a body of which the parts are movable amongst each other with entire facility; and that the mathematician must trace the consequences of this condition upon his equations. This accordingly was done, by the founders of mechanics, both for the cases of the equilibrium and of motion. Newton's attempt to solve the problem of the *figure of the earth*, supposing it fluid, is the first example of such an investigation: and this solution rested upon principles which we have already explained, applied with the skill and sagacity which distinguished all that Newton did.

We have already seen how the generality of the principle, that fluids press equally in all directions, was established. In applying it to calculation, Newton took for his fundamental principle, the equal