

teenth vibration of the other, while all the intermediate vibrations of the two tones would, in various degrees, disagree with each other; and thus every such cycle, of fifteen and sixteen vibrations, might be heard as a separate beat of sound. Now, Sauveur wished to take a case in which these beats were so slow as to be counted,<sup>8</sup> and in which the ratio of the vibrations of the notes was known from a knowledge of their musical relations. Thus if the two notes form an interval of a semitone, their ratio will be that above supposed, fifteen to sixteen; and if the beats be found to be six in a second, we know that, in that time, the graver note makes ninety and the acuter ninety-six vibrations. In this manner Sauveur found that an open organ-pipe, five feet long, gave one hundred vibrations in a second.

Sauveur's other method is more recondite, and approaches to a mechanical view of the question.<sup>9</sup> He proceeded on this basis; a string, horizontally stretched, cannot be drawn into a mathematical straight line, but always hangs in a very flat curve, or *festoon*. Hence Sauveur assumed that its transverse vibrations may be conceived to be identical with the lateral swingings of such a festoon. Observing that the string C, in the middle of a harpsichord, hangs in such a festoon to the amount of 1-323rd of an inch, he calculates, by the laws of pendulums, the time of oscillation, and finds it 1-122nd of a second. Thus this C, his *fixed note*, makes one hundred and twenty-two vibrations in a second. It is curious that this process, seemingly so arbitrary, is capable of being justified on mechanical principles; though we can hardly give the author credit for the views which this justification implies. It is, therefore, easy to understand that it agreed with other experiments, in the laws which it gave for the dependence of the tone on the length and tension.

The problem of satisfactorily explaining this dependence, on mechanical principles, naturally pressed upon the attention of mathematicians when the law of the phenomena was thus completely determined by Mersenne and Sauveur. It was desirable to show that both the circumstances and the measure of the phenomena were such as known mechanical causes and laws would explain. But this problem, as might be expected, was not attacked till mechanical principles, and the modes of applying them, had become tolerably familiar.

As the vibrations of a string are produced by its tension, it appeared to be necessary, in the first place, to determine the law of the tension

<sup>8</sup> *Ac. Sc. Hist.* 1700, p. 131.

<sup>9</sup> *Ac. Sc. Hist.* 1713.