made to sound. His first investigations on this subject, *Entdeckungen über die Theorie des Klangs*, were published 1787; and in 1802 and 1817 he added other discoveries. In these works he not only related a vast number of new and curious facts, but in some measure reduced some of them to order and law. For instance, he has traced all the vibrations of square plates to a resemblance with those forms of vibration in which Nodal Lines are parallel to one side of the square, and those in which they are parallel to another side; and he has established a notation for the modes of vibration founded on this classification. Thus, 5–2 denotes a form in which there are five nodal lines parallel to one side, and two to another; or a form which can be traced to a disfigurement of such a standard type. Savart pursued this subject still further; and traced, by actual observation, the forms of the Nodal Surfaces which divide solid bodies, and masses of air, when in a state of vibration.

The dependence of such vibrations upon their physical cause, namely, the elasticity of the substance, we can conceive in a general way; but the mathematical theory of such cases is, as might be supposed, very difficult, even if we confine ourselves to the obvious question of the mechanical possibility of these different modes of vibration, and leave out of consideration their dependence upon the mode of excitation. The transverse vibrations of elastic rods, plates, and rings, had been considered by Euler in 1779; but his calculation concerning plates had foretold only a small part of the curious phenomena observed by Chladni;[1] and the several notes which, according to his calculation, the same ring ought to give, were not in agreement with experiment.[2] Indeed, researches of this kind, as conducted by Euler, and other authors,[3] rather were, and were intended for, examples of analytical skill, than explanations of physical facts. James Bernoulli, after the publication of Chladni's experiments in 1787, attempted to solve the problem for plates, by treating a plate as a collection of fibres; but, as Chladni observes, the justice of this mode of conception is disproved, by the disagreement of its results with experiment.

The Institute of France, which had approved of Chladni's labours, proposed, in 1809, the problem now before us as a prize-question:[4]— "To give the mathematical theory of the vibrations of elastic sur-

[1] Fischer, vi. 587.
[2] Ib. vi. 596.
[3] See Chladni, p. 474.
[4] See Chladni, p. 357.