13. Older synthetical method.

termed the synthetical method, and has in modern times survived principally in England, where inductive reasoning, based upon observation of detail, has since the age of Lord Bacon been most successfully cultivated.[1] These different ways of approaching the same subject will frequently engage my attention in the course of this survey: the greatest mathematicians of modern times have recognised the importance of both aspects, and the enormous progress of the science itself has depended, no doubt, on an alternating employment of them. Leibniz clearly foresaw this when, in his correspondence with Huygens and others, he urged the necessity of not abandoning the purely geometrical view, or entirely sacrificing the older for the modern methods.[2] There can, however, be no doubt that

[1] See on this point the opinion of an authority, Hermann Hankel, in his highly interesting and suggestive lecture, 'Die Entwickelung der Mathematik in den letzten Jahrhunderten' (Tübingen, 1869, republished by P. du Bois-Reymond, 1884). Speaking of the age of Leibniz he says: "Though on the Continent mathematicians were not so conservative as in England, where a purely geometrical exposition was considered to be the only one worthy of mathematics, yet the whole spirit of that age was directed to the solution of problems in geometrical clothing, and the result of the calculus had mostly to be retranslated into geometrical forms. It is the inestimable merit of the great mathematician of Basel, Leonhard Euler, to have freed the analytical calculus from all geometrical fetters, and thus to have established analysis as an independent science. Analysis places at its entrance the conception of a function, in order to express the mutual dependence of two variable quantities. . . . The abstract theory of functions is the higher analysis. . . . The conception of a function has been slowly and hesitatingly evolved out of special and subordinate conceptions. It was Euler who first established it, making it the foundation of the entire analysis, and hereby he inaugurated a new period in mathematics" (p. 12, &c.)

[2] To Huygens, 16th September 1679: "Je ne suis pas encor content de l'Algèbre, en ce qu'elle ne donne ny les plus courtes voyes, ny les plus belles constructions de Géométrie. . . . Je croy qu'il nous faut encor une autre analyse proprement géométrique ou linéaire, qui nous exprime directement *situm*, comme l'Algèbre exprime *magnitudinem*. Et je croy d'en avoir le moyen, et qu'on pourroit représenter des figures et mesures des machines et mouvements en caractères, comme l'Algèbre représente les nombres ou grandeurs" (Leibniz, Mathem. Werke, ed. Gerhardt, vol. ii. p. 19).