second was the invention of a new and shorter method of calculating the orbit of a planet from a limited number of contiguous observations.[1] This method was communi-

The latter was the first addition made after 2000 years to the knowledge of this matter possessed by the ancients. (See 'Disquis. Arithm.,' sec. 365: "Magnopere sane est mirandum, quod, quum jam Euclidis temporibus circuli divisibilitas geometrica in tres et quinque partes nota fuerit, nihil his inventis intervallo 2000 annorum adjectum sit," &c.; and his manuscript note to this passage, given by Schering, vol. i. p. 176: "Circulum in 17 partes divisibilem esse geometrice, deteximus 1796, Mart. 30.") It is probably owing to the independent manner in which Gauss approached the subject that he early found the necessity of treating subjects of higher arithmetic (*i.e.*, of the theory of numbers or "discrete magnitudes" as distinguished from algebra, which is the theory of "continuous magnitudes") by an independent method, for which he invented a language and an algorithm. He thus raised this part of mathematics into an independent science, on which the 'Disquisitiones Arithmeticæ' is the first elaborate and systematic treatise. Legendre's 'Traité des nombres' (1799) is a complete thesaurus of all that was at that time known and of what was added by him, but it does not attempt to establish the science on a new basis.

[1] On the 1st January 1801 Piazzi at Palermo had found a movable star of 8th magnitude, RA. 57° 47′, ND. 16° 8′, which he announced to Bode at Berlin as a comet on the 24th January; but a few days later he concluded it must be a planet, and named it "Ceres Ferdinandea." No one besides Piazzi could find the star, but several astronomers, Piazzi himself, Olbers at Bremen, and Burckhardt at Paris, tried to calculate the orbit from the observations of the discoverer, which were contained within only 9 degrees. The attempt to do so under the supposition of either a circular or a parabolic or an elliptic orbit failed, and Olbers expressed the fear that with the circular or elliptic elements which had been published in Zach's periodical, it might prove impossible to find the star when it should again become visible. Very near the expected time, as late as the beginning of December, Gauss communicated his elements to Von Zach, who published them at once, recommending astronomers to follow Dr Gauss's figures and look 6° to 7° more eastward than the positions of Burckhardt, Piazzi, and Olbers indicated. And actually on the 7th December 1801 Zach himself, and on the 1st January 1802 Olbers, succeeded in finding the star, "like a grain of sand on the sea-shore," very near the positions calculated by Gauss. These results, followed soon by the discovery of other planets by Olbers and Harding, gave a great impetus to the study of astronomy. Gauss's methods were published *in extenso* in the now celebrated 'Theoria motus corporum cœlestium' in 1809. Two problems are herein treated in a novel and complete manner. The first was to calculate by a simple and accurate method from the necessary number of observations the orbit of a planet or comet on the assumption of Newton's law of gravitation, but without any other special conditions.