of thought has shown itself in other fields of research, and led to similar innovations. I will here only mention one other line of inquiry, where neither exact nor metaphysical reasoning alone suffices, but where a combination of both is essential. I mean the gradual change which, mainly through the writings of German mathematicians, has come over our fundamental conceptions in the region of geometry, algebra, and the theory of numbers. This subject belongs so essentially to the domain of pure thought that a history of thought seems specially called upon to take notice of it. Accordingly I intend to devote a special chapter to it. At present it interests us mainly because it is an outcome of that peculiar modification which the exact or scientific spirit of thought underwent when, introduced by French and English models, it came in contact with the philosophical and classical ideal of learning in Germany. I will repeat more clearly and concisely what I mean. The exact methods of thought, mainly elaborated in France, and there largely applied, give to science its accuracy and definiteness. In spite of this accuracy and definiteness, it is not immediately clear whether they will lead to completeness of knowledge, or whether they may not be misapplied. To guarantee completeness, to make sure that in the whole great field no portion has remained untouched and unexplored, that love of detail, that searching and exploring spirit, is required which is nursed pre-eminently by historical and classical studies. And to avoid the abuse of existing methods, there is further required that critical spirit which inquires into the value of principles

41. Criticism of principles of mathematics.