mon;[1] nor should we forget the suggestive writings of George Boole.[2] The influence of these men originated outside of Cambridge, and a history of mathematics at that university does not contain their names,[3] though the ideas of which they have been the bearers have largely entered into the text-books and the teaching of the Cambridge school.

So far I have mainly dealt with one side only on which the progress of science depends, namely, the methodical use of experiment, measurement, and calculation: this

quaternions — complex quantities which are compounded of a purely algebraical or quantitative element and three distinct elements corresponding to the three directions or dimensions of space. He was the first to work out this calculus, and the labour occupied twenty years of his life. In Hamilton's calculus of quaternions, distance (or length) and direction are introduced as they naturally present themselves when we deal with geometrical or physical problems, instead of all quantities being reduced to lengths, as was the case in the Cartesian geometry. Hamilton thus broke through the conventionalism of the latter and showed how the consideration of directions in space forces us to extend the original operations of arithmetic. It is interesting to note how simultaneously Grassmann (see p. 243, note 1) in his 'Ausdehnungslehre' (1844) and Von Staudt in his 'Geometrie der Lage' (1847), quite independently worked at similar extensions of our arithmetical and geometrical conceptions, and how subsequently quaternions, in which Hamilton had seen a powerful method for solving geometrical and physical problems, present themselves as a special form of the extended algebra and geometry elaborated from these different beginnings. Whilst the practical usefulness of the calculus has been demonstrated by some extensive applications, as, for example, to spherical trigonometry, the ideas contained in it—frequently without Hamilton's notation—are gradually finding their way into text-books, and the strangeness which for half a century prevented the labours of Hamilton, Grassmann, and Von Staudt from being generally appreciated, is disappearing. A popular exposition of the relation of quaternions to general arithmetic is given in O. Stolz, 'Grössen und Zahlen,' Leipzig, Teubner, 1891.

[1] The excellent treatises of Salmon on 'Higher Algebra,' 'Higher Plane Curves,' 'Geometry of Three Dimensions,' and 'Conic Sections' have in their German translations by Fiedler done a great work in systematising and popularising modern conceptions in algebra and geometry. See Gino Loria's treatise on the "Principle Theories of Geometry" in the German translation by Schütte, Leipzig, 1888, p. 25, &c.

[2] See p. 247, note 2.

[3] See Rouse Ball, 'A History of the Study of Mathematics at Cambridge,' 1889.