scientific research are involved and opened out by this statement.

13. Lines of thought emanating from it.

First, There is the purely theoretical task of defining clearly what is meant by the different words which are used, and which in the formula are expressed in algebraic symbols. What is the definition of force, what of mass, what of distance? The 'Principia' give Newton's definitions.[1]

Second, The definitions must be given in such a way that they express definite measurable quantities; and in order to verify and apply the formula, methods must be devised for measuring these quantities as they occur in nature, and these measurements must be actually carried out.[2]

[1] It will be readily admitted that the definition of force as measured by change of motion, and the definition of mass as the quantity of matter, are definitions involving some difficulty. As to distance, it may be thought that this is a purely mathematical, not a physical quantity. So it would be if physical bodies were mathematical points, such as the planets in a first approximation may be considered to be. But in comparing the attraction of the earth upon a body at its surface with that on the moon, the dimensions of the earth could not be neglected, and the problem presented itself how the quantities of mass and distance, in the case of the earth and the body on its surface, had to be defined. It appears from a statement by Prof. Glaisher (see Rouse Ball, 'History of Mathematics,' p. 297, &c.) that the publication of the 'Principia,' containing the gravitation formula, was delayed, because Newton found it difficult to prove that in a sphere the different parts with their different distances from any point need not be considered separately, but that a quantity equal to the whole mass situated at the centre of the sphere may be substituted. Laplace showed a century later that this property of the sphere exists only for one decreasing function of the distance—*viz.*, that of the inverse duplicate ratio. It exists likewise for that function which increases in proportion to the distance, but for none other (see 'Principia,' 1st ed., pp. 198, 200; 'Mécanique céleste,' 1st ed., vol. i. p. 143). Hitherto the delay in publishing the 'Principia' was (see Brewster, 'Life of Newton,' vol. i. p. 290) always attributed to the erroneous figure of the moon's distance from the earth, with which Newton had been reckoning, and which did not satisfy the gravitation formula.

[2] Up to the beginning of this century the merit of carrying out accurate measurements of astronomical constants is about equally divided between France and Eng-