of the degree of approximation which we can attain to. And this does not only refer to the methods of calculation which we adopt,—is not only a consequence of the limits of our mathematical powers; this element of error attaches likewise to our actual observations, to the imperfection of our senses and of our instruments. The many sources of mistake and inaccuracy which surround us may either combine to produce an absolutely useless result, or may be adroitly adjusted so as very largely to destroy each other.[1] The arrangement of instruments of observation and calculation, so as to minimise our errors, is a special branch of science. Before the time of Newton few minds

is the same as that of finding the centre of gravity of a number of weighted points. This centre has the property that the sum of the squares of its distances from these points is a minimum. After the method had been introduced, Laplace and Gauss independently tried to prove it by a variety of considerations. These have not always been accepted as conclusive, though it is remarkable that very different ways of attacking the problem all lead to the same result, and that the rule is confirmed by actual trials on a large scale. It has been shown that the method of least squares in the case of a series of observations of one and the same quantity is equal to taking the arithmetical mean,—a process which recommends itself to common-sense, though it is not easy to prove it mathematically to be the best. On the whole, the calculus of probabilities and the so-called law of error are attempts to put into figures and mathematical formulæ a few common-sense notions, and it is interesting to see to what complicated processes of reasoning a combination of these simple notions may lead. The literature of the subject, belonging almost entirely to this century, is very large, Laplace and Gauss heading the list. Encke has summarised the scattered discussions of Gauss and Bessel in his memoir on the subject, reprinted in Taylor's 'Scientific Memoirs' and in the 2nd vol. of Encke's 'Abhandlungen,' Berlin, 1888. De Morgan, Airy, and Jevons ('Principles of Science,' vol. i.) in England have done much to popularise the subject, and Bertrand ('Calcul des Probabilités,' 1888) has very fully discussed the principles of the whole matter and shown up the weak points. The application of the calculus to statistics will occupy us in a future chapter.

[1] Not only has every instrument its constant errors, but even every observer himself has what is called a personal equation—*i.e.*, he is subject to constant errors of observation, dependent on the peculiarity of his sense organs, or his temperament, &c. This was hardly recognised at the beginning of this century, when Maskelyne, the Astronomer Royal, dismissed an assistant whose observations showed a constant difference from his own.