two properties of the gravitation formula have been brought out by a long line of investigations, carried on with the view of substantiating or of refuting the formula. They mark the development of whole sciences, the foundation of quite novel branches of research. I propose briefly to follow up these developments.

20.
Matter and force mathematically defined.

Common-sense has never had any difficulty in knowing what matter and force are, or in defining them for the purposes of practical life. But it took thousands of years to find a definition of these quantities which could serve as the basis of ·exact measurement, and permit calculations of results into which both factors entered in varying

recent publications that attention is sufficiently drawn to the fact that very few mathematical formulæ in physics or chemistry are more than approximations. The law of gravitation is one of the few mathematical expressions which, besides being universal, have stood the most rigorous tests as to accuracy. A most interesting attempt to prove the inaccuracy of Newton's law was made, but speedily abandoned, by Clairaut, one of the earliest Newtonians in the old Academy of Sciences. Clairaut began about 1743 to study the lunar theory in the light of Newton's system, which Madrin before him ·had already despaired of reconciling with the facts of observation. When he himself, on calculating the annual motion of the moon's apogee (or farthest point in its orbit round the earth), found only half the value which observation furnished, he was tempted in his communication to the Academy of November 1747 to suggest that the Newtonian formula might require a correction for great distances. This suggestion was followed, as Lalande tells us, by a veritable scandal in the learned world. Buffon, for purely metaphysical reasons, objected to this infringement of the simplicity of the laws of the universe. The opponents of Newton's system had a short triumph, which however was speedily reversed when Clairaut, putting a greater precision into his calculations by taking inequalities into account which he had previously neglected, explained to the Academy in May 1749 that he had succeeded in reconciling the movement of the moon's apogee with the law of attraction according to the inverse square of the distance. From that time the Newtonian theory, to which only shortly before mathematicians like Euler had been won over, reigned supreme. See Lalande in the 4th volume of Montucla's 'Histoire des Mathématiques,' p. 67, &c. Euler's merits in solving many problems in physical astronomy were so great that the Academy procured permission from Louis XV. to receive him as a *surnuméraire*, the eight places granted to external members being all occupied.