

29.
Geometrical
axioms.

I may, in passing, mention here that in the course of our century certain views have been put forward in pure mathematics, or rather in geometry, which make it conceivable, if not probable, that our ideas of space might not apply to immeasurably small or to immeasurably large dimensions.¹ Should the future progress of thought

Miething, 'L. Euler's Lehre vom Aether,' p. 30). In quite recent times a similar position has again been taken up by Paul du Bois-Reymond in his essay "Ueber die Unbegreiflichkeit der Fernkraft," in the 'Naturwissenschaftliche Rundschau' (vol. iii. No. 14), and in his posthumous work, 'Ueber die Grundlagen der Erkenntniss in den exacten Wissenschaften' (Tübingen, 1890), in which he adds action at a distance as a third "ignorabimus" or unknowable problem to the two given in his brother Emil's address, "Ueber die Grenzen des Naturerkennens" (1872, reprinted in 'Reden,' vol. i. p. 105). On the Continent, about thirty years ago, the fruitlessness of pursuing this problem seemed generally admitted. Helmholtz in 1847 speaks of the initial assumption "that all actions in nature are to be reduced to attracting and repelling forces, whose intensity depends merely on the distance of points mutually acting on each other" (*actio in distans*), and Du Bois-Reymond repeats this in 1871 in his address. But it is significant that Helmholtz, who (through his memoir on vortex motion in 1858) gave such an impetus to the mechanical explanations of molecular forces, modified his views on this point (see his address on Magnus, 1871, 'Vorträge und Reden,' vol. ii.); accordingly in the reprint of his memoir of 1847 he has accompanied it with some significant remarks on the necessity of that initial assumption (1881, 'Wissen-

schaftliche Abhandlungen,' vol. i. p. 68).

¹ Reimann was probably the first to give expression to this line of thought. His memoir on this subject, "On the Hypotheses which lie at the Foundation of Geometry," bears the date 1854. It was read before the Philosophical Faculty of Göttingen in the presence and at the request of Gauss, on whom it made a profound impression (see the biographical notice on Reimann by Dedekind, attached to Riemann's 'Gesammelte Werke,' Leipzig, 1876). The memoir was not published till after Riemann's death in 1867. In England the late Prof. Clifford introduced the subject to the Cambridge Philosophical Society in 1870: "The axioms of plane geometry are true within the limits of experiment on the surface of a sheet of paper, and yet we know that the sheet is really covered with a number of small ridges and furrows, upon which these axioms are not true. Similarly although the axioms of solid geometry are true within the limits of experiment for finite portions of our space, yet we have no reason to conclude that they are true for very small portions; and if any help can be got thereby for the explanation of physical phenomena, we may have reason to conclude that they are not true for very small portions of space" (see Clifford's 'Mathematical Papers,' p. 21. Compare also his lectures on "The Philosophy of the Pure Sciences" in 'Lectures and Essays,' vol. i. p. 295 sqq.)